Exercise 25

For the following exercises, determine the domain and range of the quadratic function.

$$k(x) = 3x^2 - 6x - 9$$

Solution

Any value of x can be plugged into a polynomial function, so the domain is

$$\{x \mid -\infty < x < \infty\}.$$

In order to determine the range, first write the quadratic function in vertex form by completing the square, which makes use of the following algebraic identity.

$$(x + B)^2 = x^2 + 2xB + B^2$$

Factor the coefficient of x^2.

$$k(x) = 3(x^2 - 2x - 3)$$

Notice that $2B = -2$, which means $B = -1$ and $B^2 = 1$. Add and subtract 1 on the right side and then use the identity to make x appear in only one place rather than two.

$$k(x) = 3[(x^2 - 2x + 1) - 3 - 1]$$

$$= 3[(x + (-1))^2 - 4]$$

$$= 3(x - 1)^2 - 12$$

Because the coefficent of the squared term is positive, the parabola opens upward; in other words, the squared term takes on values between zero and infinity. The smallest value of $k(x)$ is $0 - 12 = -12$, and the highest value of $k(x)$ is $\infty - 12 = \infty$.

$$\{y \mid -12 \le y < \infty\}$$